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Symmetry breaking for ratchet transport in presence of interactions and magnetic field

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We study the microwave induced ratchet transport of two-dimensional electrons on an oriented semidisk Galton board. The magnetic field symmetries of ratchet transport are analyzed in presence of electron-electron interactions. Our results show that a magnetic field asymmetric ratchet current can appear due to two contributions, a Hall drift of the rectified current that depends only weakly on electron-electron interactions and a breaking of the time reversal symmetry due to the combined effects of interactions and magnetic field. In the latter case, the asymmetry between positive and negative magnetic fields vanishes in the weak interaction limit. We also discuss the recent experimental results on ratchet transport in asymmetric nanostructures.

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I. I. INTRODUCTION

The appearance of a directed flow induced by a zero-mean monochromatic force is a generic nonequilibrium phenomenon known as the ratchet effect (see e.g. [1–3]). The ratchet transport at nanoscale attracts a significant interest in last years (see [4] and Refs. therein). The electron ratchet currents induced by a monochromatic *ac*-driving have been experimentally observed in asymmetric mesoscopic structures at high [5], very low [6] and GHz [7] frequencies. In the later experiments the ratchet effect was visible even at room temperatures but these experiments were lacking a detailed analysis of magnetic field resonance effects, typical of two-dimensional electron gas (2DEG) in a magnetic field [8, 9] and a dependence study of ratchet current on microwave polarization. We note that the presence of resonances in a resistivity dependence on a magnetic field and polarization dependence of ratchet transport ensure that a voltage induced by a microwave radiation appears due to an asymmetry of lattice structure and not due to other asymmetries potentially present in an experimental devices.

The theoretical studies of 2DEG deterministic ratchet transport on a semidisk Galton board has been started in [10] and further extended in [11, 12],[13, 14]. The theoretical studies show that the ratchet effect exists not only for 2DEG but also for electrons in graphene plane with oriented semidisk lattice [15]. The numerical simulations and analytical theory developed in [10, 11],[12, 13],[14, 15] have been done for noninteracting electrons. They established that a directed chaotic deterministic ratchet transport emerges in such asymmetric nanostructures due to microwave radiation. The direction of ratchet current can be controlled by the radiation polarization. The theoretical studies show that the ratchet effect also exists in a presence of moderate and strong interactions between electrons [16].

The theoretical works initiated the detailed experimental studies of the chaotic deterministic ratchet 2DEG transport on the semidisk Galton board of antidots performed by Grenoble group [17]. These experiments clearly demonstrated the existence of ratchet transport in a high mobility 2DEG based on AlGaAs/GaAs heterojunstions with semidisk array. The polarization dependence of ratchet current is found to be in a qualitative agreement with the theory dependence for noninteracting electrons. It is also experimentally shown [17] that the ratchet is absent in arrays with circular antidots ensuring that the effect is produced by semidisks and not by always present device asymmetries. More recently the Grenoble group performed ratchet experiments with 2DEG in Si/SiGe heterostructures [18, 19] where the interaction effects between electrons are expected to play more important role [14]. The characteristic feature of these experiments is the dependence of ratchet transport on a magnetic field in presence of interactions. Thus it is important to understand the properties of ratchet of interacting particles in a magnetic field which creates a symmetry breaking in space and time. We note that previously the theoretical investigations were done only for noninteracting electrons in a magnetic field [12, 14] or for interacting electrons without magnetic field [16]. Thus in this work we perform a more general study analyzing the properties of ratchet transport in presence of interactions and magnetic field.

Indeed, an applied magnetic field induces a chiral movement of charge in a conducting sample. This chirality may be revealed in optical measurements such as the Faraday effect [20, 21]. An emergence of a static magnetization due to an *ac*-electric field is known as the inverse Faraday effect (see e.g. [22]). Hence, one can expect that transport properties of a chiral structure will strongly depend on the sign of magnetic field. However, this argument, based on spatial symmetries, should also take into account that the equilibrium transport measurements are also constrained by the time reversal symmetry that implies Onsager-Casimir reciprocity

relations [23, 24]. Due to these relations a two terminal conductance is always symmetric with magnetic field masking the chirality. However, since the time reversal symmetry is valid only for equilibrium samples, it can be destroyed for measurements in a nonlinear transport regime. The appearance of magnetic field asymmetry in a nonlinear two terminal transport, has attracted a significant theoretical and experimental attention in the mesoscopic physics community. Indeed, a microscopic disorder potential has no symmetry, and the absence of self-averaging in coherent samples makes such investigations possible. Surprisingly, even if all symmetries are broken in the out of equilibrium regime theoretical calculations predict that interactions are required to observe a magnetic field antisymmetric component in nonlinear conductance. While measurements on coherent samples seem to support these predictions, measurements on samples with artificial asymmetric structures may produce asymmetric (or even antisymmetric) nonlinear transport even in regimes where interactions do not seem to play a role (for e.g. high density) [17, 25]. Thus, the investigations of ratchet transport in presence of interactions and magnetic field will allow us to analyze the Onsager-Casimir reciprocity relations in a new frame.

In order to gain a deeper insight on the symmetry properties of nonlinear transport, we investigate numerically an interacting two-dimensional electron gas (2DEG) with a periodic array of asymmetric (semidisk) antidots oriented in a preferential direction using the square lattice of oriented semidisks discussed in [15, 16]. In this model a semidisk of radius r_d is placed in a square of size R and then this square covers periodically the whole (x, y) plane (see inset of Fig. 1). We consider a homogeneous monochromatic linearly polarized electric field $\mathbf{E} = E_o \cos\omega t(\cos\theta, \sin\theta)$. This field creates a rectified current flow due to the asymmetric structure of the antidot superlattice. The direction of the flow can be controlled by the polarization of the microwave field and by a magnetic field perpendicular to the 2DEG plane.

The quantitative description of the ratchet current can be obtained on the basis of kinetic theory [14]. However this theory rapidly breaks down in presence of a magnetic field since it does not capture the particle dynamics when cyclotron radius becomes of the order of the antidot radius. Moreover it was shown recently [16] that interactions could strongly modify the rectified current, hence this system allows to investigate in detail the role of interactions on the magnetic field symmetry properties of the nonlinear response. In fact, we show that the antisymmetric component of the rectified current can be splitted in two terms. One term is weakly dependent on interactions and can be interpreted as a Hall drift of the rectified current in the preferential direction fixed by the semidisk superlattice. The second term vanishes in absence of interactions as predicted by theories [26],[27],[28]. Hence, depending on the measurement geometry, the antisymmetric component of the rectified current may vanish or not in absence of interactions.

II. II. MODEL DESCRIPTION AND RESULTS

In this work we consider a 2DEG with elastic semidisk scatterers of radius r_d, oriented in direction $\mathbf{e}_x = \mathbf{x}$, and placed in a periodic square lattice of size $R \times R$ (see inset of Fig 1). The electron motion is affected by an electric microwave field $\mathbf{E}\cos\omega t = E_0 (\cos\theta, \sin\theta, 0)\cos\omega t$ linear polarized at angle θ to $\mathbf{e}_x$, and a transverse, uniform and constant magnetic field $\mathbf{B} \propto \mathbf{e}_z/R_L$ (inversely proportional to the Larmor radius R_L). The system also interacts with a Nosé–Hoover thermostat which equilibrates the ensemble of particles to the Boltzmann distribution with temperature $T = mv_T^2/2$ in a characteristic time τ_H (see e.g. [29]). The electron interactions are treated in the frame of the mesoscopic multi-particle collision model proposed by Kapral (see e.g, [30]). The method consists in dividing the coordinate space of each square of size $R\times R$ with N particles in N_{cel} collision cells. Inside each cell the collisions are mimic with a rotation of all particle velocities on a random angle in the moving center-of-mass frame, preserving the total momentum and energy of the system. These rotations are done with a repetition period given by a characteristic time τ_K. In this way large and small values of τ_K correspond to weak and strong interactions respectively. In this work we follow our previous studies of interactions effects on ratchet transport [16] where the interactions were treated in the frame of Kapral approach.

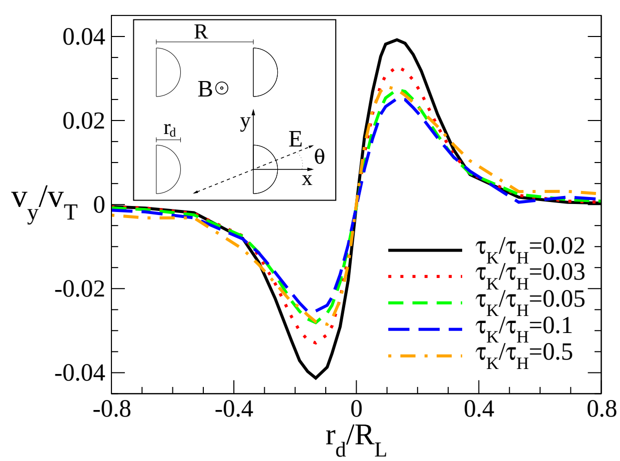

FIG. 1: Dimensionless average ratchet velocity v_y/v_T in direction y as a function of a magnetic field B given by dimensionless parameter $r_d/R_L \propto B$. The electric field is linearly polarized in y axis ($\theta = \pi/2$) with amplitude and frequency $E_0 = 0.1$ and $\omega = 0.1$, and the temperature of Hoover thermostat is $T = 1$ with $\tau_H = 10$. The interactions between electrons goes from strong interaction regime ($\tau_K/\tau_H = 0.02$) to weak interactions ($\tau_K/\tau_H = 0.5$) as it is shown on legend. The inset panel shows the directions of electric and magnetic fields in a general case, and the distribution of semidisk periodic array with an orientation direction in x axis. The geometric ratio is fixed at $R/r_d = 4$ as it is shown on the inset.

We fix the geometric ratio $R/r_d = 4$ in order to work at low antidot density and to avoid geometrical particularities which may exist for $R/r_d \simeq 1$. The number of particles is fixed at $N = 10^4$, and for numerical simulations we choose dimensionless parameters $T = 1$, $r_d = 1$, $\tau_H = 10$, and electron charge and mass $e = m_e = 1$. The Kapral grid for interactions is fixed to have $N_{cel} = 100 \times 100$ cells in the whole space region R^2. Therefore the only parameter controlling the interaction strength is the inverse Kapral time $1/\tau_K$. This choice of parameters is similar to those used in [16]. We remind that τ_H determines the relaxation time to the equilibrium.

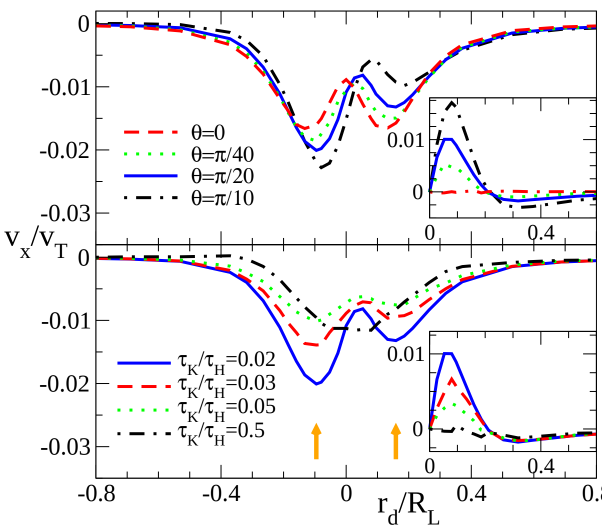

FIG. 2: Dimensionless average ratchet velocity in direction x (v_x/v_T) as a function of magnetic field. *Top panel* shows the strong interactions regime (with $\tau_K/\tau_H = 0.02$) for the rest of the parameters specified in caption of Fig 1. The electrical field is polarized near x axis, where dashed red curve represents the symmetrical case of $\theta = 0$. The small deviation angles are shown at $\theta = \pi/40$ by dotted green curve, $\theta = \pi/20$ by solid blue curve, and $\theta = \pi/10$ by dot-dashed black curve. *Bottom panel* shows the case of fixed polarization angle $\theta = \pi/20$, and different strength of interactions between particles. The interaction times are $\tau_K/\tau_H = 0.02$ (solid blue curve), $\tau_K/\tau_H = 0.03$ (dashed red curve), $\tau_K/\tau_H = 0.1$ (dotted green curve) and $\tau_K/\tau_H = 0.5$ (dot-dashed black curve). *Inserts* in both panels show the dimensionless asymmetry $[v_x(r_d/R_L) - v_x(-r_d/R_L)]/v_T$ as a function of r_d/R_L for the same parameters marked by color. Orange arrows show the values of magnetic field analyzed in Fig. 3.

The steady-state net current of the system is proportional to the average particle velocities. This current can be described as a vector $\mathbf{j}$ splitted in two components of different magnetic field symmetry: $\mathbf{j} = \mathbf{j}_s + \mathbf{j}_a$ where $\mathbf{j}_s$ are symmetric and $\mathbf{j}_a$ antisymmetric components. These two vectors depend quadratically on the alternating electric field amplitude $\mathbf{E}$. and can also depend on $\mathbf{e}_x$ and $\mathbf{B}$, therefore their most general expression reads:

$$\begin{aligned} \mathbf{j}_s &= f_1(B)\,(\mathbf{e}_x \cdot \mathbf{E})\,\mathbf{E} + f_2(B)\mathbf{E}^2\mathbf{e}_x \\ \mathbf{j}_a &= g_1(B)\,(\mathbf{e}_x \cdot \mathbf{E})\,(\mathbf{B} \wedge \mathbf{E}) + g_2(B)\mathbf{E}^2\,(\mathbf{B} \wedge \mathbf{e}_x) \end{aligned} \quad (1)$$

Here $f_i(B)$, $g_i(B)$ $(i = 1, 2)$ are functions of the modulus of the magnetic field $B = |\mathbf{B}|$ depending implicitly on the rest of model parameters. Specifically, we focus on the dependence of the antisymmetric component of the current on the strength of interactions between electrons.

We note that both terms of second line of Eq. 1 can be decoupled for different values of θ. For electric and magnetic fields used in this model, the dimensionless velocity can be written as

$$\begin{aligned} \frac{v_x(\theta)}{v_T} &= \frac{E_0^2}{\sqrt{2T}}\left[\tilde{f}_1\cos^2\theta + \tilde{f}_2 - \frac{\tilde{g}_1\sin 2\theta}{2}\right] \\ \frac{v_y(\theta)}{v_T} &= \frac{E_0^2}{\sqrt{2T}}\left[\frac{\tilde{f}_1\sin 2\theta}{2} + \tilde{g}_1\cos^2\theta + \tilde{g}_2\right] \end{aligned} \quad (2)$$

where $\tilde{f}_i$ and $\tilde{g}_i$ $(i = 1, 2)$ are symmetric and antisymmetric functions of the magnetic field in $\mathbf{z}$ direction (or related Larmor radius $R_L = v_T/B$). These functions are proportional to the functions in Eqs. 1 $\tilde{f}_i \propto f_i(B)$ and $\tilde{g}_i \propto g_i(B)/R_L$ (where we take $R_L > 0$ and $R_L < 0$ for magnetic field in $\mathbf{z}$ and $-\mathbf{z}$ directions respectively). Following Eqs. 2 we can note that for $\theta = 0$ and $\theta = \pi/2$ velocities in x direction are symmetric while the y-components are antisymmetric.

In the case of polarized electric field in y direction ($\theta = \pi/2$), v_y can be asymmetric in the magnetic field only due to the second term of second Eq. 2 noted as $\tilde{g}_2$. This case is illustrated in Fig. 1 for the dimensionless quantities v_y/v_T plotted versus r_d/R_L at different values of Kapral time τ_K and the rest of parameter specified in the caption. The data of Fig. 1 show that the dependence of ratchet velocity v_y/v_T is an asymmetric function of $r_d/R_L \propto B$ with maximum of $|v_y|$ at $r_d/R_L \approx \pm 0.15$. The amplitude of this maximum increases by a factor 2 with the increase of interactions (decrease of τ_K).

Following the first line of Eq. 2 the asymmetry given by $\tilde{g}_1$ can be analyzed via parameter dependence of v_x/v_T at small angles of linear polarization of electric field directed both along $\mathbf{x}$ and $\mathbf{y}$. The emergence of $\tilde{g}_1$ from a symmetric behavior in the magnetic field is shown in top panel of Fig. 2, for v_x/v_T with small values of θ in the strong interactions regime ($\tau_K/\tau_H = 0.02$). In the case of $\theta = \pi/20$, bottom panel of Fig. 2 shows the behavior of asymmetry in v_x/v_T for different interaction times, going from $\tau_K/\tau_H = 0.02$ to $\tau_K/\tau_H = 0.5$. The striking feature of bottom panel of Fig.2 is that the asymmetry in a magnetic field is rather strong at strong interactions while in the limit of weak interactions the asymmetry completely disappears and we recover the symmetric curve as a function of a magnetic field.

A deeper analysis of the symmetry properties can be done by considering the flux of velocities in coordinate space. It is known that for interacting particles the average velocity behavior can be rather complex with an

emergence of some vortexes [16]. In Fig. 3 we present the velocity flux for weak and strong interactions regimes with positive and negative magnetic fields. The analyzed values of Larmor radius correspond to the relative minima of v_x/v_T marked with arrows in Fig. 2 at negative and positive values of r_d/R_L. Fig. 3 shows the flow structure at $r_d/R_L \simeq 0.16$ on top panels and at $r_d/R_L \simeq -0.1$ on bottom panels. The polarization angle in four panels is fixed at $\theta = \pi/20$, and therefore the reflection symmetry $y \to -y$ is not preserved. The data show that the vortex asymmetric structure is more pronounced in the case of strong interactions on right panels.

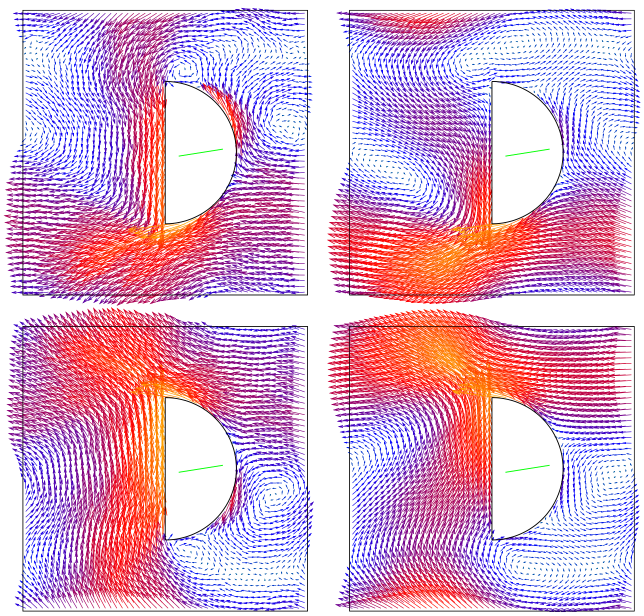

FIG. 3: Map of local averaged velocities in (x, y) with $x, y \in [-3, 3)$ and linear polarized electrical field $\theta = \pi/20$ (shown by green line inside semidisk scatterer); other parameters are the same as in Fig. 1. The left panels show the cases of weak interactions between particles at $\tau_K/\tau_H = 0.5$, while the right panels show strong interactions regime at $\tau_K/\tau_H = 0.02$. The values of magnetic field are given by $r_d/R_L = 0.1591$ on top panels, and $r_d/R_L = -0.0955$ on bottom panels (pointed with orange arrows in Fig. 2). The velocities are shown by arrows which size is proportional to velocity amplitudes, which is also indicated by color [from yellow (gray) for large to blue (black) for small amplitudes].

The polarization dependence of ratchet current in $x-$direction is analyzed in Fig. 4. We see that even for various magnetic fields the dependence on polarization angle θ is essentially symmetric at weak interactions (top panel) while at strong interactions (bottom panel) we have a strongly asymmetric behavior at moderate magnetic fields ($r_d/R_L = 0.053, 0.133$). At a relatively large magnetic field ($r_d/R_L = 0.53$) the amplitude of ratchet current becomes rather small since the Larmor radius becomes smaller than the distance between semidisks. Thus

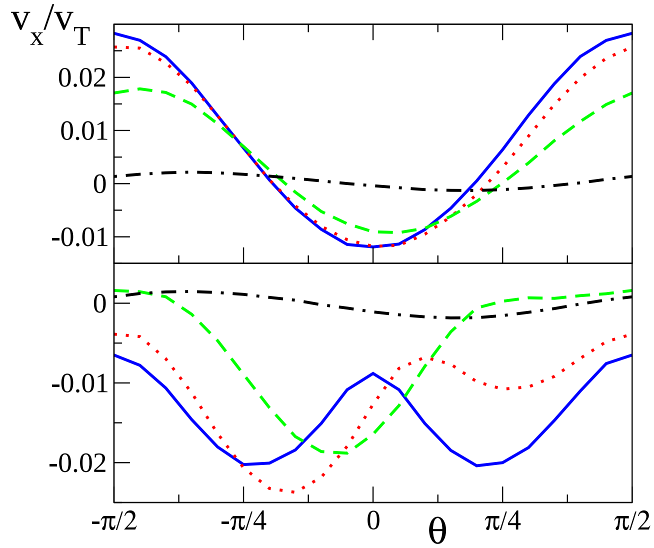

FIG. 4: (color online) Polarization dependence of x-component of the averaged ratchet velocity v_x/v_T, for weak interactions between particles in top panel ($\tau_K/\tau_H = 0.5$), and for strong interactions in bottom panel ($\tau_K/\tau_H = 0.02$). Four different values of magnetic fields are shown in both panels: zero magnetic field ($r_d/R_L = \infty$, solid blue curve), $r_d/R_L = 0.053$ by dotted red curve, $r_d/R_L = 0.133$ by dashed green curve and $r_d/R_L = 0.53$ by dot-dashed black curve.

the data of Fig. 4 also show that the asymmetry of ratchet transport appears only in a presence of interactions.

III. III. DISCUSSION

In this work we study the symmetry properties of the ratchet transport on an oriented semidisk antidot superlattice. We show that while the ratchet flow on the oriented semidisk superlattice (along $x-$axis) does not depend on the sign of magnetic field for a noninteracting 2DEG, interactions can give rise to an antisymmetric component of the flow in the semidisk direction as a function of a magnetic field. This result is consistent with the case of mesoscopic samples where deviations from Onsager-Casimir reciprocity relations were shown to occur only in presence of electron-electron interactions [26],[27],[28]. On the contrary, the flow perpendicular to the semidisk direction ($y-$axis) is asymmetric even when interactions are absent. We argue that the origin of this component of the flow is a Hall drift of the rectified current. This contribution is absent in disordered mesoscopic samples because the translational symmetry is broken by a disorder potential where no preferential direction is present for the flow.

The comparison with [17],[18],[19] highlights various aspects of these skillful experiments. At weak electron-electron interactions typical of AlGaAs/GaAs hererojunctions [17] there is a qualitative agreement between

the theory and experiment on polarization dependence and approximately symmetric current response on sign change of a magnetic field, even if the absolute quantitative values differ from theoretical predictions (see [16] for a more detailed discussion). For experiments with Si/SiGe heterostructures [18, 19] the effects of interactions are stronger and asymmetric response in a magnetic field is clearly observed in experiments (see e.g. Fig.3 in [18]). In this case the change of polarization from $\theta = 0$ to $\theta = \pi/2$ does not change the sign of photovoltage that also appears in theoretical models at strong interactions as discussed in [16]. Also the ratchet effect is clearly suppressed in experiment and theory at large magnetic fields. However, at the same time there are significant differences between experiments [18, 19] and theoretical results presented here. Indeed, for $\theta = \pi/2$ the theory predicts a change of the photovoltage sign upon inversion of the magnetic field (see Fig. 1 above) while there is no such sign change in experiments (see e.g. Fig.3 in [18] and in [19]). It is possible that finite sample size leads to a certain charge accumulation in the experimental setup (see indications for that in Fig.4 in [19]) that may explain the difference with the theory where the analysis is done for an infinite lattice size. We should also note that the present studies were done for a square lattice of semidisks while in the experiments [17–19] were performed on an hexagonal lattice of semidisks. However, in both cases the density of semidisks is relatively low (since $R/r_d = 4$ here and $R/r_d \approx 5$ in [18, 19]). Thus, the ratchet transport is created mainly by scattering on a one semidisk (see discussion in [14]) and the difference between square and hexagonal lattices is not expected to be important.

The discussed caparison between the present theoretical studies and the most advanced experiments reported in [17],[18],[19] shows that the further experimental and theoretical research of the electron ratchet transport in asymmetric nanostructures with dynamical chaos represents a significant fundamental scientific interest.

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